THINKING ABOUT THINKING

CASSIUS J. KEYSER

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BY

CASSIUS J. KEYSER, Ph.D., LL.D

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CONTENTS

PAGE

I IS ALL THINKING ORGANIC BEHAVIOR? 1

II AUTONOMOUS OR POSTULATIONAL THINKING . . . 17

III. THE IMPORTANCE AND AVAILABILITY OF AUTONOMOUS THINKING 37

IV DETECTION OF POSTULATES . . 73

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THINKING ABOUT THINKING

THINKING ABOUT THINKING

I

IS ALL THINKING ORGANIC BEHAVIOR?

Readers of this essay are invited to think a little about thinking. Psychologists have not yet been able to tell us what thinking is. But most knowledge is knowledge of the undefined as knowledge of love, for example, or knowledge of life; and as for thinking, though we can not define it, we know enough about it to know that there are many kinds. I wish to notice some of them, some of their similitudes and differences, the rôles they play, the services they render, the various ranges of their significance.

From among the various kinds of thinking I have selected one kind for special consideration because it seems to surpass all the others in power and importance. It may be appropriately called autonomous thinking for a reason to be given at a later stage. Strangely enough, autonomous thinking is, of all kinds, perhaps the least familiar. Yet it is so rooted in the nature of intellect that any one having talents above the level of a moron's can be led into a fair understanding of it and be disciplined in a measure to follow its ways; it is known to be essential to the greatest achievements of science; it is available in all fields of thought and, because it is so, is destined, I believe, to attain pre-eminence among the intellectual and scientific agencies which, it is hoped, will become more and more effective in determining the future course and char-

acter of human history. It is the principal aim of this little book to sketch the nature of that unique type of thinking, to signalize its importance, to indicate some of the chief results it has already produced, and thereby, if possible, to increase the number of those who employ it habitually in all the affairs and fields that demand or challenge their thought. Nowadays historians sometimes speak of the history of history. Logicians might speak in a similar sense of the logic of logic. The following discussion is not submitted as a contribution to that difficult subject. It is far less fundamental, less hard to understand and is, I venture to hope, a little less arid.

Some years ago my wife and I were walking westward along a street of New York against the slanting rays of a beautiful evening sun. Suddenly my companion stopped, shrank back,

blanched and veered as if confronted by the spectacle of some horror. A forward glance instantly revealed the cause: a large white dog viciously pursuing diagonally across the street a handsome kitten running its best for life. The direction of the kitten's going indicated a somewhat distant goal. My immediate impulse to rush to the kitten's aid was instantly checked by the fear that an effort to help might result in hindering. And so I stopped, stood still and watched. I regret that it is impossible for me to convey in words an adequate sense of the marvelous little drama enacted there before my eyes. Yet the reader, if attentive and sympathetic, will be able, I hope, to realize the scene worthily in imagination. Just as the kitten reached the middle of the street the dog had overtaken and was about to seize it. What happens? Quick as a flash the

kitten wheels about, rears upon its hind feet, ears laid back flat, eyes blazing like those of a maddened hawk defending its young, strikes the dog's nose again and again, dodges, side-steps, retreats, advances, keeping always upon its hind feet, strikes again, quick as lightning leaps to a point at the dog's side, thence upon its back, runs forward and down between its ears over its face, on reaching the nose springs from it upward and outward in a kind of spiral curve, so turning in mid air as to land in fighting posture face to face with the enemy, strikes again, swiftly repeating all the tactics employed before, thrice performing the entire program in what seemed scarcely more than a minute. At this stage of the battle it seemed to me that the dog was a bit confused and disconcerted. So, too, it may have seemed to the kitten. At all events the latter sud-

denly turns, makes again for its home in a basement, the dog pursuing, both rush down the steps and disappear. I follow quickly, fearing the worst. Behold! The kitten has escaped by a corridor, entered a room, come to the front, and is now pressing its little white breast against the iron grating of an open window, triumphantly striking out at the nose of its angry, puzzled defeated foe.[1]

Why have I told that story? I have done so because the kitten's behavior is an admirable example of what I shall call organic thinking. Why 'thinking'? And why 'organic' thinking? I call its behavior thinking because the kitten conducted a campaign—a defensive and offensive campaign—such that, had it been thoroughly thought out in advance,

[1] For another purpose I have told this story in my *Science and Religion,* Yale University Press.

any military expert, were he to study the plan carefully, the general features of it and its many details, and especially if he had been permitted to witness its brilliant execution, would pronounce it, plan and performance alike, a master piece of military intelligence or military genius. And I call it *organic* thinking because the behavior in question was organic behavior, not an inorganic thing's mechanical reaction to the impact of a mechanical force, but the action of an organism responding to stimulus and responding thereto as a living whole. What of the dog's behavior? Was that, too, a specimen of organic thinking? It was; not indeed so fine a specimen as the kitten's but not less genuine. But, you may wish to say, there was a very important difference between the two behaviors. That of the kitten was successful—it was an achievement, a triumph; the dog's was

not. True. But successfulness is not an essential quality of organic thinking. On the contrary, all such behavior, indeed all thinking, is an adventure; it essentially involves an element of chance, an experimental element. An experiment, however, is not something that must succeed, it is a thing to be tried. To speak of guaranteeing in advance the issue of an experiment does violence to the very conception of experiment; it is like speaking of a round square or a footless tripod or the cerebral cortex of an acephalous creature; that is to say, it is a contradiction in terms, a sound without sense.

It will be noted that in the foregoing discussion certain old familiar terms are conspicuous by their absence. There is no mention of 'soul,' for example, nor of 'mind' nor of 'consciousness.' Why not? The answer is a logical one—the

terms were not needed. Not being needed they are excluded by Occam's Razor: *Entia non sunt multiplicanda praeter necessitatem.* Some one, however, will say: "You have indeed employed the term, organic thinking, and it seems to me that speech about thinking must always involve at least tacit or implicit reference to soul." Well, let us return for a moment to our kitten and have another look at the facts. In watching it we are watching a living thing—a thing whose nature it is to act; we observe the response of a living organism to stimulus—the stimulus of a menace to life; we witness a certain series of co-ordinated acts—a series constituting behavior, organic behavior, behavior of an organism. Such behavior I have called organic thinking. In so doing, I have simply given a name to a fact—the fact being the behavior. What was it

that did the behaving? The kitten. What, then, was it that did the thinking? Same question, same answer: the kitten. What we have here is evident—just a few familiar facts of observation, the lawful naming of one of them, a twice-stated question, its evident answer, and that is all—nothing to bewilder, no trick, no legerdemain, no mystery, no reference to soul. If, on the other hand, we were to say that the thinking in question—that is to say, the behaving—was not really done by the kitten but was done by some mysterious invisible intangible metaphysical ghost called a feline soul, it is evident that we should be thus leaping into a pit of darkness. And the pit is bottomless because, having once taken the plunge, we could never stop if we were logically consistent. For we should then have to invent a new soul, soul of the feline soul, to account

for the latter's behavior, and then, for a like reason, a soul of the soul of the feline soul, and so on endlessly—an infinite regressus of metaphysical rot. So much for the hypothesis of a feline soul. Do I deny that the kitten had mind? I neither deny it nor assert it. I do deny that, if the kitten had mind, its mind was one thing and the kitten another. I assert that, if the kitten had mind, the mind it had was kitten. What of being conscious? Did the kitten have a conscious desire to save its life? Its conduct was indeed such as to incline one to believe, especially at first, that it had such a conscious desire. I have, however, no sufficient ground for asserting that it had or that it had not.

Is all thinking organic thinking in the indicated sense of the term? It may be such. It is about as certain as any thing can be that all the thinking which has

occurred in the world, conscious or subconscious, human or subhuman, logical or hypological, profound or shallow, sane or insane, was done by living organisms. There is no convincing evidence to the contrary. And so it may be that all sorts of thinking whatsoever, a mollusc's or a man's, a kitten's or a king's, a philosopher's or a pig's, that of an Einstein or that of an earth-worm, have in common the property of being literally organic behavior, the action of some living organism responding as such to stimulus.

Suppose that what is thus seen to be possible were found to be a fact. What would be the effect of the discovery? I mean, of course, the effect upon our thinking, upon our behavior, for the discovery, once it were made, would be a part of our environment and would bear to us the relation of a stimulus to a liv-

ing organism. What would our response—our behavior—be? Would we behave as if thinking, on becoming identified with a kind of organic behavior, had thereby lost its dignity? We might thus behave if we happened to be very stupid specimens of the human type of living organisms. But if we were worthy specimens — specimens worth preserving—, we would behave, not as if the dignity of thinking had been lost, but as if that of organic behavior had been found. It is evident that, after the supposed discovery, the character or nature of thinking would be precisely what it had been, despite the improvement in our understanding of it. What was admirable in it would remain none the less admirable on being found to belong to organic behavior. The familiar distinctions and qualities, as logical and illogical, formal and informal, deduc-

tive and inductive, empirical and apodictic, critical and creative, clear and obscure, coherent and loose, simple and profound, all of these and their kind would have to be regarded as distinctions and qualities pertaining to the characteristic behavior of living organisms but they would not thereby suffer any impairment of interest, validity or value. The difference between the thinking of an ameba detecting, pursuing, capturing its prey and the thinking of a Socrates discoursing upon immortality or that of an Aristotle investigating the laws of logical thought; the difference between the thinking of a mouse learning by trial to thread a simple maze for a piece of cheese and the laboratory thinking of a Pasteur or that of Galileo seeking to ascertain by experimental observation the laws of falling bodies; the difference between the thinking of a Darwin motiv-

ated by scientific curiosity, by disinterested love of truth, and "wishful thinking" like that of a "fundamentalist" striving to preserve the precious consolations of myth or that of a snake trying to catch a toad for a meal; the difference between the thinking of a measuring worm slowly, tentatively, humbly crawling from point to point of its little world and that of a human geometrician engaged in exploring the infinite zones of hyperspace: all such contrasts and diversities of type and scope and complexity and freedom and power, however vast and striking, would remain undiminished in magnitude and impressiveness. They would indeed make manifest the infinite variety in what would be thenceforward the most marvelous thing in the world—Organic Behavior—the response of living things to the varying conditions, the changing needs and the advancing ideals

of life. It may be added that then and thereafter the proper subject matter of human psychology would be known to be, not the inscrutable ways of intangible ghosts, but the observable behavior of men, women and children.

II

AUTONOMOUS OR POSTULATIONAL THINKING

Its Nature and Great Achievements

Happily neither the validity nor the significance of the following discussion depends upon any answer or settlement of the question raised in the foregoing section, whether every kind of thinking is, in the sense there indicated, a type of organic behavior. Whatever the right answer may be, we know that kinds of thinking differ immensely in manifold ways. Of all the kinds the one that, in virtue of its intellectual quality and scientific power, least resembles the organic type exemplified by the ameba

(say) or by our now familiar kitten is the If-Then kind, distinguished from all others by its characteristic form: if so-and-so, then so-and-so. A thing so remarkable ought to have a name. For the sake of convenience we must give it one. On account of its form it might, with fine propriety, be called hypothetical thinking were it not for the fact that this phrase has acquired a somewhat different and perhaps inferior connotation. I shall call it autonomous thinking or postulational thinking. The propriety of the designation will appear presently.

What is meant by autonomous or postulational thinking? How may we know whether, when we are thinking, our thinking is of the kind in question? If we omit certain niceties[1] as inessential to

[1] For a deeper view see my *Mathematical Philosophy,* E. P. Dutton & Company.

the purpose of this essay, the answer, though it is exceedingly important, may be brief and fairly easy to understand. It runs as follows:

Suppose that some sufficiently important subject in no matter what field, science, it may be, or philosophy or ethics or economics or jurisprudence or politics or sociology or education, has so strongly appealed to your curiosity and so long engaged your attention that you are moved to construct a doctrine of the subject for the guidance of yourself and others or because of the joy that doctrinal building gives. If you are at all qualified to do autonomous or postulational thinking, you will have logical sense enough to know at least three things, one of them relating to definition of terms, one of them to proof or demonstration of propositions and the third to clarity and cogency of discourse.

You will know that no doctrine can, without committing the unpardonable sin of circularity, undertake to define all of the terms it employs but that every doctrine must employ one or more terms regarded as being, without definition of them, sufficiently intelligible for the purposes of clear discourse. You will know that for a like reason no doctrine can furnish proof of all its propositions but that every doctrine must contain one or more propositions which it takes for granted, using them without demonstrating them. And you will know that a doctrine can have maximum clarity and cogency when and only when it has the minimum of undefined terms and undemonstrated propositions. If, knowing these things, you begin the work of construction by deliberately selecting (1) the few terms or ideas which you intend to employ without defining them, (2)

a small set of compatible propositions which must of course connect the terms already chosen and which you intend to employ as a basis without demonstrating them; and if, having performed that initial task of selecting what we may call your undefined terms and undemonstrated propositions, you proceed with the task of definition and proof, defining all other terms as you have occasion to use them, and demonstrating the propositions that are logically implied by, and deducible from, the undemonstrated ones: if you do that, if in the work of building your doctrine you proceed in the manner I have indicated, then and only then you are engaged in autonomous or postulational thinking in the full sense of the term. I call such thinking postulational because the propositions that are taken for granted in a reasoned discourse, though they are sometimes called

assumptions or primitive propositions or axioms, are most commonly called postulates; and I call it autonomous thinking because a doctrine constructed by it is an autonomous doctrine, one, I mean, that is independent in the sense of being a logically organized whole containing as an integral part of itself the foundation on which it stands.

I have said that such thinking is characterized by its form and that its form is the If-Then form. We may readily see that such is the case. For the postulates are not asserted and they need not be true; they are employed as premises or impliers, as hypotheses, as logical *ifs*. Neither are the derived propositions asserted and they need not be true; they present themselves merely as implicates, they are logical *thens*. What *is* asserted and what is true is that, *if* the postulates are true, *then* their implicates are true.

It is important to grasp the fact that this *if-then* assertion is true both when the postulates are true and when they are false. It is, moreover, the only kind of assertion that occurs in autonomous thinking, and that is why I have said that the characteristic form of such thinking is the If-Then form.

Is this manner of thinking a new thing in the world or is it old, as old perhaps as the race of man? And is the capacity for it a common endowment or an endowment of only a few? Were we to consider similar questions regarding scientific procedure in general or regarding an art, as music, for example, or painting or architecture or sculpture, it would be necessary to distinguish between what is rudimentary and what is mature, and we should be obliged to render the answer in terms of evolution. The same is true in the present case. As

a type of activity postulational thinking is so distinctively human and is so helpful in all human affairs, however commonplace and lowly, that we must *suppose* it was done in some small measure, in a haphazard and rudimentary way, probably unconsciously, by all men and women since the advent of man, just as today we *see* it everywhere done in some small measure, in haphazard and rudimentary fashion, for the most part unconsciously, even by the immense herd of those whose knowledge of science is nil. But as a thing come to maturity, as a full-grown procedure deliberately and systematically employed in the construction and criticism of doctrine, postulational thinking, far from being a thing of ancient date, is a strictly modern activity, and, far from being engaged in by the many, the great work accomplished by it has been the work of a few.

I have said that, as a ripened form of human activity, postulational thinking is strictly modern. Indeed its very earliest outstanding achievement was produced only twenty-two centuries ago—hardly more than a yesterday in the long backward stretch, a half million years or more, of human time. With that achievement most readers have some acquaintance for I am, of course, referring to Euclid's *Elements*. I need not describe it for its general make-up is familiar: a large body of propositions of which a few are taken for granted and the rest are demonstrated—deduced, that is, from the former ones by the processes of logic. It is the most famous example of autonomous thought in the history of science. The fact that its subject-matter is geometric is not essential, it is accidental, an accident of history. What is essential is the method of think-

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ing, the postulational method, and this, as I have already said, is available in every field of thought. As a *model* of such thinking the *Elements* of Euclid is somewhat imperfect. It errs by excess and by defect: by excess of definition, defining or attempting to define, certain terms which it nevertheless employs, quite rightly, as terms undefined; and by defect of postulation, in failing to state explicitly certain propositions which it nevertheless employs, quite unconsciously, as postulates. But in spite of its imperfections the model in question is astonishingly fine—so fine indeed that its chief significance could not be grasped by the declining philosophy of its time. For when it was produced it was so incomparably superior to any other product of human thinking that men were dazzled by it, blinded by its very brilliance, so much so that, though they

admired it and in a sense understood it, they failed to perceive that its chief significance was, not geometric, but methodological; they did not perceive that, though the matter of the *Elements* was special, the method was not but was perfectly general, equally applicable to all kinds of scientific matter. So skillful was the application in this instance that matter and method seemed inseparable; if they were indeed two, they were but two interlacing aspects of one and the same thing—and that thing was just geometry. And so it happened that the first great product of postulational thinking was destined to remain long without a rival in kind. The famous model of such thinking, despite its vitality and power and beauty, was to remain long barren—barren, I mean, in the sense of failing to engender its kind in any subject, mathematical or non-mathe-

matical—and many centuries were to pass before philosophy or science could produce a second great example of autonomous thought.

To Spinoza belongs the credit of having been the first important thinker to see clearly that the method of Euclid's *Elements* was far more general than its matter and of having been the first to attempt application of the method to matter of another kind. Ethics and Geometry—what possible subjects can differ more widely than these in their essential natures, in the character of their contents, in the type of ideas involved? Nevertheless Spinoza deemed it possible to construct by the method in question a body of etchical doctrine having the same validity as that which he ascribed to the geometry of Euclid, and he bravely essayed the task. He did not succeed but his failure was a glorious

failure, which none but a great man can make.

Is it, then, impossible to construct sound ethical doctrine by the method of postulational thinking? I hope to show at a later stage that it is not impossible. Why, then, did Spinoza fail? He failed because he endeavored to construct an ethical theory that was to have absolute, universal and eternal validity, the kind of validity which he, like all other thinkers of the time, ascribed to the geometry of Euclid but which, we know today, neither geometry nor any other doctrine, regarded as describing the actual world, can have. It thus appears that postulational thinking has one essential quality which the philosophy of the time had not discerned. For universal, absolute and eternal validity could not have been ascribed to the *Elements* of Euclid or to any other doctrine

based upon postulates unless these had been regarded, not as mere assumptions, which they are, but as statements of known fact, which they are not. The time was not yet ripe for the production of a second great work by the postulational method. For that it was necessary to wait for the coming of Saccheri who in 1733 produced by the method in question the first non-Euclidean geometry ever constructed. Here at length was a work worthy to be called the second great example of autonomous thought for, though the subject matter was again geometric as in the case of Euclid's *Elements*, the doctrine itself was non-Euclidean, a fact serving to show that the postulational method was available for the construction of differing doctrines. But Saccheri's work had no influence because it remained unknown. It seems that even the author himself

never became aware of the nature of his great achievement. And so the work had to be done again and it was, but not before the lapse of another hundred years. The second doing of it was attended with better fortune. For the non-Euclidean geometries created independently and almost simultaneously by the Hungarian, John Bolyai, and the Russian, Lobachevski, were so shocking to the regnant philosophy of the time as not only to challenge attention but to compel it and they were at the same time so invulnerable that logicians, mathematicians, physicists and philosophers were obliged to scrutinize as never before the method by which the strange new doctrines had been constructed. There ensued a revolution in the theory of knowledge, the philosophy of science was greatly advanced, and postulational thinking was henceforth to play

increasingly a double rôle, that of builder and that of critic or judge.

Since Lobachevski and Bolyai gave their brilliant creations to the world a century has elapsed. It has been a scientific century. As the years have gone by worthy emulators of the mentioned pioneers have appeared in ever increasing number. Thinking of the kind represented by them is still very, very far from having become a common activity of their fellow men and women; it has not yet become a common habit even of scientific men; but a vastly greater amount of it has been done in the last hundred years, indeed a far greater amount of it has been done by the present generation, than by all the preceding generations of the human race. A most astonishing fact and most heartening too, if we but pause to contemplate it. We have seen that the first 500,000 years of

human existence failed to produce one great example of autonomous thought; we have seen that 2000 years ago there was but one such example; we have seen that 100 years ago there were but two of them. And what do we see now? We see that in the years of the present generation scientific doctrines of the autonomous type have risen by the score, the most imposing and most beautiful edifices in the modern City of Science. It seems worth while to point out a few of them.

The elliptic geometries of Riemann and the late Felix Klein ought perhaps to be mentioned first on account of their fame as companions and rivals of Euclid's *Elements* and the non-Euclidean doctrines of Lobachevski and Bolyai. For another example, there stands what is perhaps the most beautiful of all mathematical edifices, projec-

tive geometry—child of architecture—a science born of an art. There, too, stand Plücker's line geometry and the sphere geometry of Lie—two four-dimensional geometries sprung from conceiving space, not in the traditional way as an ensemble of points, but as a plenum of lines in the one case and a plenum of spheres in the other. These and many other geometric doctrines, including manifold theories belonging to hyperspace, have been constructed, in recent years or else reconstructed, by postulational thinking and will stand forever among the monuments of autonomous thought. If we turn from the field of Geometry to that of Analysis (as mathematicians call it), we find strikingly exemplified the same type of achievement. The theory of the real variable (or the real numbers); the theory of the complex variable (or complex numbers);

the theory of abstract systems known as groups; *Mannigfaltigkeitslehre* — the theory, that is, of manifolds or aggregates—which subtly pervades the whole domain of mathematics like an omnipresent ether: these and kindred doctrines have now been cast in the If-Then form and join in witnessing to the constructive and critical power of postulational thinking. It is a grave mistake to believe that the availability of postulational thinking is confined to mathematics. It is not so confined. The method is available in every field of thought, in the physical sciences, in the moral or social sciences, in all matters and situations where it is important for men and women to have logically organized bodies of doctrine to guide them and save them from floundering in the conduct of life. With this question of availability I shall deal at a later stage.

For the present it will be sufficient to point out that the method of postulational thinking, though hitherto most of its triumphs have been in mathematics, has already invaded out-lying fields. It has invaded the great field of physics where it has given autonomous form to the classical theory of rational mechanics, has placed the special theory of relativity upon a postulational base, and is in process of advancing the general theory to the same estate. And the social sciences, too, are being invaded by it. It has not yet constructed there any autonomous doctrine of outstanding importance but it is everywhere present as a method of criticism and is working great transformations in economics, for example, in ethics, in theology, and the philosophy of law.

III

THE IMPORTANCE AND AVAILABILITY OF AUTONOMOUS THINKING

The Relation of Autonomous to Empirical Thinking

In the preceding chapter I have briefly described the nature of postulational thinking, outlined the course of its development as a scientific method, shown the increasing rapidity of its progress in recent years, exemplified the variety and character of its achievements, asserted its availability for all fields of thought, and emphasized its importance. At this

point a critical reader may desire to raise a question or two.

"I now understand fairly well," he may say, "the nature of postulational thinking; the history of its development is interesting; the rapidity of its recent advancement is a striking phenomenon; the doctrines which it has made autonomous constitute an impressive array: all this I grant. But I question the alleged availability and the alleged importance of the method. In respect of availability I note that the autonomous doctrines which you have cited belong either to the domain of mathematics or to that of physics. Among them there is no representative of the moral or social sciences. Whether the method in question can be successfully applied to the subject matter of these sciences, whether it is possible to erect in one of them, in ethics, let us say, or in economics, a scientific

edifice having the structure and form of an autonomous doctrine, I seriously doubt. With reference to the question of importance I note that many of the autonomous theories which you have cited as showing the power of postulational thinking did not originate in such thinking but were well established, effective, flourishing doctrines before they were made autonomous, some of them long before. Of such doctrines projective geometry is one example; the theory of the real numbers is another; rational mechanics is a third; and there are many others. I therefore question whether the postulational method really is so very important after all."

I rejoice in the intelligence of my skeptical critic for the considerations he has understandingly adduced in support of his doubt are not easier to understand than those which ought to dispel it. The

questions he has raised are two: Is the method of postulational thinking available for all fields of thought? And is the method highly important? Though they are not independent, the questions are distinct. An answer to either of them does not necessarily imply an answer to the other. A method may be very important without being widely available or widely available without being very important. Yet it is obvious that, other things being equal, the importance of a method depends directly upon the range of its availability. And so it is seen that our critic's questions, though quite distinct, are not independent. They may be considered conjointly. In the discussion to follow I shall so consider them, dealing now with one and now with the other, as occasion serves. I have answered both of the questions in the foregoing chap-

ters. The answers are affirmative—confident and emphatic. It remains to justify them.

To justify the answers it is necessary to consider the manner in which postulational thinking is related to thinking of another type—a very common and very comprehensive type which I shall call empirical. What do I mean by empirical thinking? I shall employ the term to denote every variety of thinking which, in respect of its logical quality, falls between organic thinking as exemplified by our kitten fighting the dog and autonomous thinking as exemplified by Euclid constructing his *Elements*. The range of variation is vast, having for its lower limit a kind of thinking in which logic plays no part and for its upper limit a kind in which the rôle of logic is dominant and supreme. It is thus seen that empirical thinking is

characterized by two properties or facts: (1) in any specimen of it there is present some ingredient or process of logic; (2) the occurrence of the logical factor, however frequent or important, is always subordinate, its presence being that of a servant, not that of a sovereign.

Why call thinking of the type thus characterized empirical? What is the propriety of the description? It is not found, as one might suppose it to be, in the fact that all thinking of the type involves empirical elements; for, although such elements are indeed common to the type, they are not characteristic of it. There is something empirical in the thinking of the kitten—animals learn to do things by a kind of experimentation, by *trying* to do them. In autonomous thinking, too, empirical processes occur at every stage: postulates and undefined terms must be selected for a certain pur-

pose—the selection involves experimentation; the postulates have to be tested for compatibility—such a test is experimental; and experimentation is again involved in the devising of proofs or demonstrations. And so the question recurs: why have I applied the adjective empirical to the type of thinking in question? I have done so because the type so described is the type of thinking which yields what it has long been customary to call empirical knowledge or empirical science.

Properly speaking, thinking is human or subhuman according as it involves or does not involve the presence of concepts—according, in other words, as it contains or does not contain some tincture of logic. The type of empirical thinking is, then, like that of autonomous thinking, a type of human thinking. But not all of the thinking done

by a human being, however brilliant he be, is human. On the contrary a very great deal of it is subhuman, being simply organic response without any logical ingredient; it is thinking of the feline or canine or ameboid type, neither rational nor irrational but subrational, neither logical nor illogical but hypo-logical. There is nothing astonishing or shocking in this. It is only a generally neglected instance of the familiar fact that the life of a higher organism and that of a lower one have constituents in common. In accord with the indicated meanings of our terms we may now say summarily: (1) if thinking be autonomous or empirical, it is human; (2) if it be human, it is autonomous or else empirical; (3) if it be subhuman, whether done by animals or by human beings, it is neither autonomous nor em-

pirical; and (4) if it be neither of these, it is subhuman.

Of human thinking, properly so called, there are, then, two great types—the empirical and the autonomous or postulational. How is the latter related to the former? That, as I have said, is a question we must consider if we are rightly to estimate the availability and the importance of postulational thinking. The type of empirical thinking is very comprehensive. In point of bulk or amount as distinguished from quality, it embraces nearly all the human thinking that has been done in the world for it includes, as we have seen, all that was done during the first half million years of human existence and all but a small fraction of what has been done since postulational thinking first appeared upon the scene a few years ago in the

guise of Euclid's *Elements*. One can not reflect upon it even a little without perceiving that the type is indeed immense. Fortunately I need not here attempt to analyze or describe it minutely. It will be sufficient to remind ourselves of a few of its familiar factors: observation, for example, experimentation, collection of facts or data, formation of concepts, manipulation of these in some incomplete measure by the processes of logic, making of guesses or hypotheses respecting causes and orders of events, testing of such conjectures by various devices for verification: all motivated partly by curiosity, partly and mainly by the felt needs of life. It is thus evident that empirical thinking is universal in the sense that it occurs in the lives of all human beings, whatever their gifts, interests or occupations. And its issue is many forms of wisdom. It gives the farmer his em-

pirical knowledge of agriculture, the sailor his empirical knowledge of navigation, the physicist his empirical knowledge of physics, the teacher his empirical knowledge of education, the moralist his empirical knowledge of ethics, the carpenter his empirical knowledge of building, the biologist his empirical knowledge of the organic world, and so on and on: *all* empirical knowledge, including all the branches of empirical science, is, as I have said, the fruit of empirical thinking.

Such knowledge is precious in manifold ways, immeasurably precious, but it does not satisfy. We crave more of it—more understanding of the same kind—and we crave something else—understanding of a different kind. I hope to show that to both of these natural cravings postulational thinking is a perfectly natural and very important response.

Empirical knowledge is expressed in various ways, as in forms of institutions, in mores or social customs, in habits, in practical procedures, but the chief of all the ways is the articulate way of propositions. It is ultimately, or tends to be, the all-comprehensive way. For such knowledge continues to shape and reshape itself till it wins at length fairly complete expression in the form of propositions, not propositions logically deduced from a set of initial premises, but empirical propositions, deposits of experience, articulate precipitates of empirical thinking.

Now consider some field or branch of empirical knowledge, say physics, practical arithmetic, ethics, the rude geometry of the carpenter, education, economics, medicine, biology, municipal law, no matter what. We sometimes say that such and such branches are new.

Of course they are not. All branches are old, as old as the race of man. In any such field or branch there has come to be an ensemble of propositions, seldom precise, rightly or wrongly taken for true. Whether precise or not, whether true or false, they serve to express what is regarded as knowledge—empirical knowledge slowly accumulated little step by step in the course of ages of empirical thinking. The propositions are empirical propositions. They constitute an empirical doctrine. At least one such doctrine for each field or branch of empirical knowledge. Consider one of the doctrines. Denote it by *D*.

Sooner or later, at some stage of *D's* development a remarkable thing happens: some one perceives that two of the propositions composing *D*, say *A* and *B*, are so related that *A* implies *B*, that *B* is a consequence of *A*, that, if *A* is true,

B is true, that *B* can be inferred or deduced from *A*, and that, therefore, supposing *A* to have been discovered first, knowledge of *B* might have followed immediately by means of pure logic instead of having to wait for a day or a century or five hundred years for the independent discovery of *B* by empirical means. That simple insight is a great event. Prior to it *D* was only an ensemble of propositions existing side by side but now they begin to wear an aspect of logical interdependence. The insight extends and multiplies itself. It is apt to be found that among the propositions in *D* there are two or more small sets from each of which another set, usually a larger one, can be inferred. And so the process of logical organization goes on, slowly and haltingly for centuries, till the advent of some one with genius enough to ask the question: Is there

among the propositions of the doctrine *D* a small set of compatible propositions from which all the other propositions of *D* can be logically deduced? Or the more general question: Is there a small set of compatible propositions (all of them or only some or none belonging to *D*) from which all of *D's* propositions, except such as may happen to belong to the set, can be logically inferred? In asking either of those questions and trying to answer it, in propounding and trying to solve the problem involved, our hero is engaged in postulational thinking in the full sense of the term. His thinking is ultra-modern, it is strictly up-to-date. He will very probably find it necessary to change somewhat the statement of the problem because in scrutinizing as he must the propositions of *D* he will almost certainly find that some of them must be rejected as false and that

the remainder require restatement in the interests of clarity. This will be a notable service rendered incidentally by postulational thinking. But to simplify our further discussion I will suppose the problem to remain, as indicated, in its obvious form.

Suppose the problem solved. What is the achievement worth? Much in many ways, as a little meditation will suffice to show. Note what has happened. Prior to the achievement we had —and, of course, we still have—the doctrine *D* setting forth in its way our empirically gained knowledge of the peculiar subject matter of some chosen field. In addition to *D* we now have, as the result of postulational thinking in response to needs left unsatisfied by *D*, a new doctrine, one of different type, an autonomous doctrine, dealing with the same matter. This doctrine which

we may denote by D', is highly important on many accounts.

It is important philosophically for it shows that the empirical knowledge slowly accumulated by untold centuries of experience in a certain field can be brought into the perfect form of a logically organized body of propositions. And thus an old philosophic dream regarding all fields is realized in one of them, for D' is an accomplished fact. If, in the light of future experience, some of the propositions of D' were seen to be untrue, it would be necessary to construct a new autonomous doctrine if we desired to have one giving a true account of that part of the actual world with which it dealt. But the actual world is only a fragment of the infinitely greater world of possibility, and D', being an indissolubly interlocked system of propositions, would, even in the case supposed,

remain an eternally valid partial account of the world of possibility.

Not only is D' thus philosophically important as a highly significant contribution to kenlore, or the theory of knowledge, but it is also important esthetically for it affords in the contemplation of it the peculiar delight of a great work of art. An autonomous doctrine is indeed a work of art of the highest order. Not of sensuous art: for it is not heard, like poetry and music; nor seen, like painting, sculpture, and architecture. Its inner harmony and its qualities of structure and form are perfect but they are revealed only to the infinitely delicate sensibility of the pure understanding; it is a work of art but the art is supersensuous.

Criticism, it has been said, is the life of science. We may almost say that criticism is the life of life. It certainly

is the life of empirical thinking, without which life could not rise above the dull level of mere monotonous repetition—doing always the same things and doing them in the same way. And now I maintain that for the great work of doctrinal criticism our autonomous doctrine *D′* or its like is of the highest importance both as instrument and as model or norm. What I am about to say in this connection respecting the importance of postulational thinking will incidentally throw light upon the general availability of the method. We know that all men and women do more or less empirical thinking about a great variety of subject-matters, ethical, religious, theological, educational, economical, political, and so on. The result is many empirical doctrines in many fields of thought. Any one of the doctrines, if fairly well developed, is a joint product of many

minds. Every human mind is a doctrine factory but most of the factories are so poorly equipped that the doctrines produced by them are very rudimentary or embryonic. A mind that is too feeble to invent doctrines itself always adopts doctrines invented by others. Consequently we all of us hold a large variety of doctrines regarding a large variety of subjects. We value them partly as helping us to live well, partly as in a measure gratifying our desire for knowledge of the world, but mainly as creeds that we are bound to make prevail. To make them prevail we resort to every conceivable means: rational and irrational, savage and civilized, brutal and humane, force, fear, flattery, bribery, threats, ostracism, prayer, preaching and teaching. Yet most of the doctrines which we thus hold as sacred creeds and solemnly urge upon the world are unin-

telligible, vague, incoherent, ignorant, shallow, silly—logically rotten.

The remedy? The remedy is criticism. But what kind? I will try to indicate. Here is a host of books each of which sets forth some empirical doctrine for the immediate enlightenment and the ultimate salvation of the world. I select one of the books almost at random and, supposing that you have been fairly well bred in the ways of postulational thinking, I request you to examine the book carefully and to give me your estimate of it. If you comply, what will your criticism be? It will run as follows:

"The author," you will say, "employs a large number of seemingly important terms and he does so with quite remarkable facility. But I have been unable to ascertain with anything like precision the sense in which he uses them. Indeed he

appears to employ one and the same term now in one sense, now in another, and now in a third without being aware of any change. Regarding the function and importance of definition he seems to be entirely innocent. He is apparently unaware of the radical distinction between defining and describing. I find that here and there he does indeed try to make some of his terms tell the meanings of others, and that is, of course, legitimate; but to my great surprise, I see him trying at a later stage to make the latter terms tell the meanings of the former ones; quite unconscious of the fact that he is thus executing a movement like that of a kitten pursuing its tail—only the kitten is playful but the author is solemn. He fancies that his book is reasoned discourse; yet he does not know that in any such discourse the terms employed fall into sharply dis-

criminated classes: a small class of terms deliberately chosen for use without definition and a larger class whose terms are defined by means of the former ones. If he does know it, the fact is concealed. Similar confusion reigns in the matter of propositions. He makes what he calls arguments, gives what he calls proofs, offers what he wishes to be taken for demonstrations. In all this many propositions are employed, no doubt unconsciously, without being stated; others appear now as if basic and later as if derived. I find that the discourse is, strictly speaking, without foundation and without superstructure. The author means well but his work is a whirling cloud of words. It stirs the emotions and it sells but as an intellectual performance, as a giver of light, it belongs to the literature of *bla, bla, bla.*"

That is what you will say. You have judged an average exposition of an empirical doctrine by the principles and the standard of autonomous thought. The judgment is severe. Is conformation to that standard always possible or practicable in expounding or advocating an empirical doctrine? If I ask you that question I know what your reply will be. You will say that in the present state of knowledge it is not always possible and that even if it were, it would often be impracticable. You will say that you are insisting upon the standard as an *ideal* which expounders and advocates ought to understand and ought constantly to own as their guide; for just in so far as human discourse departs from it, it sinks toward the level of mere chattering—the noise and gabble of our prehuman ancestors. And your reply is right.

Regarding the more strictly scientific importance of postulational thinking I will submit two considerations. The first one is that a well-wrought autonomous doctrine, the product of such thinking, is the most perfect specimen of science at its best. Indeed the autonomous form is the ideal to which every scientific branch, whether of the so-called natural or physical sciences or the so-called moral or social sciences, consciously or unconsciously aspires and to which it more and more approximates as it grows in logical maturity, becoming more self-critical, more philosophic and more artistic. I will not enlarge upon this matter here as I have dealt with it elsewhere.[1] At present I wish to make it clear that postulational thinking contributes a great deal to the advancement

[1] *The Human Worth of Vigorous Thinking.* Columbia University Press.

of empirical thinking whenever any field of the latter is entered by the former. Let us revert to our doctrines D and D', the former empirical, the latter autonomous, both of them dealing with the matter of a chosen field. It is essential to bear in mind that D' is composed of two sets of propositions, a small set whose propositions, being taken for granted, are employed simply as postulates, and a larger set whose propositions are implicates of the postulates and in D' are deduced from them by the processes of logic. It is also essential to note and keep in mind that all of the propositions in D are contained in D' and that with little or no exception their status in D' is not that of postulates but that of deductions therefrom nor that of empirical propositions as in D but that of conclusions logically derived from premises, the postulates of D'. It has already been

remarked that the kind of knowledge or understanding represented by D, though precious, fails to satisfy. It leaves us with two cravings: we demand more understanding of the same kind and we demand understanding of a different kind. In the foregoing discussions it has been made clear, I believe, that postulational thinking in giving us doctrines like D' meets our second demand. We are now in position to see that it helps materially to meet the other demand also and does it in a way that is quite unique. Recall the fact that the postulates of D' were chosen for postulates because they were found to imply the empirical propositions constituting D. Now notice what happens. In working with the postulates, in deducing from them the propositions of D, it is quickly discovered that they imply not only these propositions but an endless number

of others. For it is well known to logicians that in general the number of implicates of a system of postulates is infinite. Thus D' gets immensely enlarged, eventually containing not only the propositions of D but a host of others besides. What of it? Well, the new propositions will be known to be true provided the postulates are known to be true, and our knowledge of the actual world will be thereby extended. But that is not the point I wish to make. The point I wish to make is as follows: If the postulates are false or not known to be true, then the new propositions in question, though logically deduced, may be all of them false or all of them true or some false and some true. In the case supposed, and it is the usual case, the problem of determining which of the new propositions are true and which are false is a problem for empirical think-

ing. Observe what postulational thinking has accomplished here. Empirical thinkers find themselves almost suddenly confronted with a body of propositions of which they had never thought or dreamed. Each of the propositions is a question, a challenge. Curiosity is aroused. Empirical research is stimulated and not only stimulated but, what is even more important, it is directed—led by definite questions given to it by postulational thinking.

I can not refrain from adding a word to signalize how handsomely empirical thinking pays its debt. I mean that empirical thinking, having received so great a benefit from postulational thinking, proceeds in its turn to contribute to the progress of its benefactor. For suppose, what is almost certain to happen, that some of the new propositions of D' are found by experience to be false.

Then postulational thinking is confronted by a new problem—that of constructing a new autonomous doctrine, one whose postulates imply all of the true propositions of D' (except such of them as may chance to be among the new postulates) but none of the false ones. And so it is seen that the two great types of human thinking, far from being mutually antagonistic, serve to stimulate and to direct one another, thus co-operating to advance human knowledge whether of the actual world or of the vaster world of possibility.

At the beginning of this chapter I promised to meet two demands: to show that postulational thinking is highly important and to show that the method is available for all fields of thought. The first demand has been met fully, I believe. The second one has been met only in part. I have indeed shown that

as a means of criticism the method in question is available in all fields including those of moral or social sciences. But to meet the second demand fully it would be necessary to show that autonomous doctrines can be established in the fields of these sciences, and that I had hoped to do. It cannot, however, be done in a few additional paragraphs, and the chapter is already grown too long. Consequently I have decided to compromise. After some preliminary remarks, concerned with ethics and designed merely for orientation, I will refer to a recently published book where the matter is discussed fully, very suggestively and very penetratingly.

Ethics has its roots in certain sentiments. What the sentiments are is indicated by such terms as right, wrong, good, bad, duty, obligation, ought, ought not. Like everything else, the senti-

ments are what they are. Wherever they occur, whenever they occur, they occur as facts, as facts of nature, children born of human experience, joint products of the world and the individual. The scientific study of ethics is the scientific study of these facts as facts, as genuine constituents of the Space-Time manifold. The sentiments fashion conduct, beget rules of behavior, engender judgments, get gradually expressed in the form of propositions. The propositions, which commonly take the shape of commands, are empirical. Do they together constitute an empirical doctrine of ethics? They do not. Why not? Because they are so incompatible that, when taken together, they present a chaos. Why thus incompatible? Because the sentiments from which they spring are neither uniform nor permanent. They differ in different individ-

uals, in different communities. They are subject to the vicissitudes of a fluctuant world, of which they are part. They are functions of time and place, of circumstance, experience, history. As they differ and change, the rules differ and change, the judgments differ and change, the propositions differ and change. It is wrong to enslave one's fellow men; the institution of slavery has produced much good in the world; it is wrong to eat meat; it is not wrong to eat meat; it is wrong for women to appear in public with their faces unveiled; to deny them that privilege is wicked and silly; sometimes it is right for the people of one state to slaughter the people of another; such killing is always as bad as murder; an eye for an eye and a tooth for a tooth; *lex talionis* is unjustifiable —if smitten on one cheek, turn the other; democracy is the best, monarchy

the worst; monarchy is the best, democracy the worst; anarchy is better than any form of government; an anarchist is a loathsome monster; humility is a virtue; humility is a sign of feebleness and moral decay; to introduce among the fundamental principles of government a provision designed to prohibit the making of a glass of beer is a high and holy achievement, worthy of a great and righteous people; such a law springs from a vicious kind of intemperance—the intemperance of ignorant fanaticism and low-flying zeal; and so on and on: a literally endless array of ethical propositions were we to assemble them from all the times and places of human life. It is evident that to call such a chaotic conglomeration an empirical doctrine of ethics would be merely to make a meaningless noise. Is it, then, impossible to construct such doctrines? I am far from

thinking so. It is possible, I believe, to construct many of them. But how? By division. Indeed such doctrines already exist, but it is necessary to identify them, to pick them out. I assume—what, I believe, will be readily granted—that no ethical proposition in the immense ensemble of them is contradicted by all the rest. I assume, in other words, that each of the propositions is consistent with a considerable number of others. And so it is theoretically possible—and is no doubt in a measure practicable—to sort the vast totality into sets—how many I know not—such that the propositions of any set are compatible with ane another. Compatibility being a logical relationship, it is evident that the propositions of such a set constitute an empirical doctrine or system of ethics and that the number of such systems is the same as the number of sets. One or more of the

empirical doctrines having been established or identified it would remain for postulational thinking to construct, in its now familiar way, corresponding doctrines of autonomous type. These would differ among themselves in the field of ethics as geometries differ in the field of their subject-matter.

Here I must bring this particular discussion to a too early termination. I do so with but little regret because I am able to refer, and have great pleasure in referring, to a beautifully written work where a reader, if he be interested, will find cogently presented the thesis regarding the availability of postulational thinking not only for ethics but for economics and other fields of social science. I allude to a book by Jacques Rueff, entitled *Des Sciences Physiques aux Sciences Morales.*

IV

DETECTION OF POSTULATES

Selection of postulates is one thing. Detection of postulates is another. When we choose a set of propositions with a view to deducing from them the propositions of some empirical doctrine, we are selecting postulates for an autonomous doctrine regarding some part of the actual world. When we choose a set with a view merely to deducing their consequences, we are selecting postulates for an autonomous doctrine regarding some part of the world of possibility. In either of these cases we are aware that we are employing postulates; we know what the postulates are for we

have deliberately selected them in advance; and at every stage we are vividly conscious of their service as guides. In empirical thinking, however, it is not so. Here, too, as we shall see, postulates are involved; but we are very often not aware of the fact and, even when we are aware of it, it remains to determine in any given case what the postulates are. In attempting such determination—in endeavoring, that is, to discover what propositions we have been unconsciously taking for granted—we are engaged in what I shall call detection of postulates. In regard to selection of postulates I have said a good deal in the preceding chapter but in regard to detection of postulates I have thus far said nothing explicitly, having decided to reserve it for the emphasis of special consideration because it is, I believe, the most significant aspect of postulational

thinking. Indeed the matter is of so great importance that I hardly know how to portray it adequately in the course of a few paragraphs. For if rightly presented it will be seen that the work of postulate detection is not an obligation peculiar to a few scientists, a few philosophers and a few other specialists but it is a universal obligation, common to all men and women; it will be seen that the work is called for, not merely in this or that specific kind of subject-matter, but in all varieties of subject-matter whatsoever; and it will be seen that it is something to be engaged in, not merely occasionally or avocationally, but habitually and regularly as an essential part of our intellectual business from day to day.

First of all it is necessary to realize—what most people do not realize because they have not been disciplined to realize

it—that in all of our empirical thinking postulates are essentially involved at every stage. The proposition, that all of our empirical thinking involves essential reference to postulates despite the fact that when engaged in such thinking people are for the most part utterly unconscious of any such reference, is very sweeping. It is indeed tremendous. Nevertheless it is true. How may we know it? We may know it by a little observation, and a little analysis, of human behavior. Every day many millions of men, women and children assert many millions of propositions, results of empirical thinking. Challenge one of the propositions and note what happens. Unless deterred by indifference or lack of time or bashfulness or fear, the assertor of the proposition defends it. How? By argument. The point to note is that, whether the argument be good

or bad, it is argument—the defense of the proposition may be feeble but the defense is argument. Now we know very well that any argument in defense or support of a given proposition, however good or bad the argument may be, makes conscious or unconscious use of other propositions and that the status of these in the argument, because they are there consciously or unconsciously taken for granted, is the status of postulates. A living organism, called a human being, asserts an empirical proposition, product of empirical thinking, receives the stimulus of a challenge or denial, makes a response which—call it what you will, organic or physiological or psychological or logical—essentially involves the use of one or more other propositions as postulates. Such human behavior is normal. What does it indicate? It indicates that in all human thinking whether empirical

or autonomous, in all our opinions and creeds, in all our discourse, spoken or written, formal or informal, sane or insane, and in all our planning, organizing, managing and conducting of practical affairs and institutions there is essentially involved explicit or implicit reference to postulates—reference, that is, to propositions which, whether formulated or not, are consciously or unconsciously taken for granted.

Suppose the generality of men and women were educated, as they might be, to realize the truth of that proposition keenly. What would be the effect of it upon their thinking? Knowing well that wherever empirical thinking occurs there are hidden postulates to be discovered, would they be eager and alert to discover them and bring them forth from their hiding into the light? When seriously engaged in thinking upon an im-

portant matter would they habitually and insistently ask themselves: What are the propositions that we are taking for granted? What are the concealed postulates upon which the validity and the significance of our conclusions depend? They would do so, I believe, as soon as they discovered that the doing of it is the sole means for meeting certain of their most deeply felt needs; and the discovery is not difficult to make. It is fairly obvious that postulate detection is demanded by the most momentous of our rational desires: the desire not to be deceived—the desire to know whether the propositions which our love of wisdom or the exigencies of life require us to consider are true or false. It is not difficult to see, but it is very important to see, that when we are bent on ascertaining which of the two qualities belongs to a given proposition, we are in every case

driven to the consideration of postulates and that when these are unknown, which is the usual case, they have to be discovered.

Take, for example, some familiar proposition of geometry, say this one: The sum of the angles of any plane triangle is equal to the sum of two right angles. If we seriously ask ourselves whether the proposition is true, we enter upon a course of reflection that runs as follows: if the proposition be true, the fact cannot be established by empirical means; for even if we could measure the angles of a triangle exactly, it would be necessary to measure the angles of an infinitude of triangles of infinitely many shapes and of infinitely many sizes from the infinitesimally small to the infinitely large; it is, of course, impossible to perform an infinite number of acts in a finite length of time, however long; but

we can not measure exactly the angles of even one triangle; the most we can do empirically is to measure inexactly the so-called angles of a few very imperfect sensuous imitations of triangles —imitations made of wire or wood or drawn by pen, pencil or chalk; abandoning empirical means as hopelessly inadequate and turning to logic for help, we may be tempted to say the proposition is true because it has been proved; we reflect, however, that the proof of the proposition consists in deducing it as a logical consequence of certain other propositions, which, as they are taken for granted, are postulates; whether these be true we know not; and so the course of reflection leads us to see that, regarding the question of the truth of the proposition, the best we can say is this: the proposition is true *provided* the postulates from which it is deduced are true.

Suppose we extend our reflection. It would go on as follows: were we to employ another system of postulates, say that of Lobachevski or that of Riemann, we would derive as consequence of the former system the proposition that the sum of the angles of a plane triangle is less than two right angles and as a consequence of the latter system the proposition that the angle sum is greater than two right angles. If we were then asked whether either of these propositions is true, we should be again constrained to reply: it is true provided the corresponding postulates are true.

What I have said here respecting familiar specimens of geometric propositions is valid for all the countless propositions of Mathematics. Not one of them has been, or can be, shown to be true by empirical means. The status of any mathematical proposition whatever

is either that of a postulate or that of a proposition deduced from some system of postulates; if in the former case the proposition be true, no one can be aware of the fact; in the latter case the best we can say is: the proposition is true if the postulates are true.

(Here I must insert a parenthesis for some pragmatist is sure to ask: can we not ascertain whether mathematical propositions are true by applying them to concrete affairs and noticing whether, when so applied, they work? The answer is no; the geometry of Euclid, that of Lobachevski and that of Riemann contradict one another at many points; nevertheless they work, and work equally well, when applied by the architect or the engineer. To try to infer that a mathematical proposition is true from the fact that it has never failed to work is to reveal a certain prac-

tician type of mind utterly incapable of distinguishing mathematics from the geometry of a carpenter or the arithmetic of a grocer.)

If we pass from mathematics to any other field, we shall find that the same enquiry will lead us to the same kind of result. Shall we try theology? Consider, for example, the proposition: Jesus was born of a virgin. Is the proposition true? We observe at once that it is not self-evident, even if we admit that there are such things as self-evident propositions, for, if it were, no one would believe it, which many do, but all would know it, which many do not. Since it is not self-evident I am not entitled to assert that it is true unless I have logically deduced it from other propositions, that is, from postulates. Even then I am entitled to say only: it is true if my postulates are true.

While fundamentalists are detecting and assembling the needed postulates and making the required deduction, I may go on to exemplify my thesis by random examples from other fields. The human race is descended from a Simian ancestry; sensation is a continuous function of stimulus; the earth was once a nebulous mass forming a part of the ancient sun; the recent world war was the result of economic causes; all natural processes are discontinuous; human beings possess immortal souls; the millionaire who bought an infinite number of pairs of boots and bought as many pairs of socks as pairs of boots bought as many socks as boots; simultaneity of events is relative, not absolute; the sense of time is only an imperfect sense of a fourth dimension of space; the chemical elements are transmutable into each other; flowers love their own beauty;

the space of the physical world is finite, not infinite; the voice of the people is the voice of god; there is a divinity that shapes our ends; and so on. But it is needless to multiply specific examples—they abound on every hand. It is better to take them by the bulk—in groups, sets or bundles. Go to a great library. There you will find many bundles of many kinds: a large bundle of astronomical propositions, a physical bundle, a socialistic bundle, various psychological bundles, communistic bundles, bundles made by advocates of democracy, economic bundles, chemical bundles, ethical bundles, theological bundles, paleontological bundles, and many others. Pick out one of them, examine it carefully, and then let me ask you a question or two. Are the propositions self-evident? If you have half the sense I credit you with, you will say "no." Have they

been strictly deduced from a definite body of premises? "I find," you will say, "more or less argumentation but of logical deduction, rightly so-called, there is little or none." Was the author, expositor or advocate a mystic claiming ability to see the truth directly, without mediation of proof? "I discovered no such claim." Did he signalize the propositions that he took for granted? "No." Was he aware that he was employing postulates? "Here and there he seemed to be somewhat aware of it but in general he was not." If he had discovered his postulates and deduced his propositions from them, would he then have been entitled to assert that the propositions are true? "He would not; he would have been merely entitled to say: my propositions are true if my postulates are true."

It is now sufficiently evident. I be-

lieve, that the task of postulate detection is urgent, immense, and omnipresent. But what is the use of working at it? If, in the case of some doctrine, we succeed by much toil in discovering the postulates, what have we gained if we do not know that the postulates are true? Well, we have found what the truth of the doctrine hinges upon, and that is certainly a gain. If, when found, the postulates must themselves be examined with respect to their truth or falsity, has not all the work to be done again? No —not done "again"; what we have to do is more work of the same kind. But if we must seek the postulates of our doctrine and then seek the postulates of these postulates, what assurance have we that the task will end? We have none. It will not end unless we arrive at a set of postulates which, when found, will be *known* to be true. That may

never happen. I rather hope it will not for it would probably be very boring to live in a world so easily exhausted. What we are seeking is truth but truth is an ideal and an ideal, if it be genuine, is not something to be realized but a thing to be pursued and approximated.

Discovery of postulates is often followed by discovery that some of them are false. What of it? The discovery that a postulate is false is a discovery—it is a step in advance, understanding is increased, we are disillusioned, knowledge has grown. When a postulate is found to be false, it has to be abandoned. The history of postulational thinking, especially the wake of postulate detection, is thickly strewn with abandoned postulates. Many a hoary one has been abandoned in recent years. In political economy the postulate of "the economic man"; in ethics the

postulate of a universal principle for the guidance of conduct; in law the centuries-old postulate of *lex naturae* or *jus naturale*, a kind of "brooding omnipresence in the sky" as Justice Holmes wittily characterized it; in physics and cosmology the postulates of cosmic absolutes—absolute space, absolute time, absolute matter, absolute natural law: these and many others have been cast overboard in our own day. Indeed a very interesting and edifying work might be written on "Abandoned Postulates." It would be a history of the progress of knowledge. For when postulates have to be abandoned, doctrines dependent on them require revision, transformation of content or form, perhaps abandonment for something better.

Postulate detection, an important aspect of postulational thinking, is not a panacea—nothing is that—but it is es-

sential as a means to many good ends: it increases knowledge in all fields; it is a powerful instrument for doctrinal criticism; it shows us how hard it is to know; it fosters scientific modesty, discourages dogmatism, favors tolerance, and makes for the maintenance and advancement of good will in the world.

THE END

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